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# GUJARAT TECHNOLOGICAL UNIVERSITY BE- SEMESTER-I \& II(NEW)EXAMINATION - SUMMER 2022 

Subject Code:3110015
Date:22-08-2022
Subject Name:Mathematics - 2
Time:10:30 AM TO 01:30 PM
Total Marks:70
Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.
Q. 1 (a) Find the Laplace transform of $t^{2} e^{-3 t}$. 03
(b) Define conservative vector field and potential function. 04
(c) Solve $y^{\prime \prime \prime}-3 y^{\prime \prime}+3 y^{\prime}-y=4 e^{x}$ using the method of undetermined $\mathbf{0 7}$ coefficients.
Q. 2 (a) Find the divergence of 03

$$
\boldsymbol{F}=\left(x^{2}-y\right) \boldsymbol{i}+\left(x y-y^{2}\right) \boldsymbol{j}
$$

(b) Find Fourier cosine integral of
(c) Integrate $f(x, y, z)=3 x^{2}-2 y+z$ over the line segment $C$ joining the07 origin to the point $(2,2,2)$.

## OR

(c) Write Green's theorem. Evaluate the integral $\oint_{C}\left\{x y d y-y^{2} d x\right\}$ where $C$ is the square cut from first quadrant by the lines $x=1$ and $y=1$.
Q. 3 (a) Obtain convolutiongoff $t$ and $e^{t}$. 03
(b) Find the Lapla d transform of $\frac{\cos a t-\cos b t}{t}$. 04
(c) Solve the infual value problem 07
$y^{\prime \prime}-y^{\prime}-2 y=0, y(0)=1, y^{\prime}(0)=0$ using Laplace transform.

## OR

Q. 3 (a) Find the inyerse Laplace transform of $\frac{s-4}{s^{2}-4}$.
(b) State second shifting theorem and find the inverse Laplace transform of04 the function $\frac{s e^{-\pi s}}{s^{2}+1}$.
(c) State convolution theorem and using it obtain the inverse Laplace
transform of $\frac{1}{s\left(s^{2}+4\right)}$.
Q. 4 (a) Solve $\frac{d y}{d x}-2 y=4-x$. 03
(b) Solve $p^{2}+2 p y \cot x=y^{2}$. 04
(c) Solve $y^{\prime \prime}+4 y=4 \tan 2 x$ using the method of variation of parameters. $\mathbf{0 7}$

OR

| Q. 4 | (a)Find particular solution of <br> $y^{\prime \prime}-2 y^{\prime}+y=\cos 3 x$. <br>  <br>  <br> (b) Solve $x^{2} y^{\prime \prime}-3 x y^{\prime}+4 y=0$ | $\mathbf{0 3}$ |
| :--- | :--- | :--- |
|  |  | $\mathbf{0 4}$ |

(c) Solve the initial value problem
$y^{\prime \prime \prime}+y^{\prime}=0$,
$y(0)=0, y^{\prime}(0)=1, y^{\prime \prime}(0)=2$
Q. 5 (a) Write Legendre's and Bessel's differential equations.
(b) Solve the differential equation 04

$$
\left(y \cos x+2 x e^{y}\right)+\left(\sin x+x^{2} e^{y}-1\right) y^{\prime}=0
$$

(c) Find the power series solution of the equation $\left(x^{2}+1\right) y^{\prime \prime}+x y^{\prime}-$07 $x y=0$ in powers of $x$.

## OR

Q. 5 (a) Write Legendre polynomials of degree one and two. 03
(b) Solve $y=2 p x+p^{2} y$. 04
(c) Solve $x(x-1) y^{\prime \prime}+(3 x-1) y^{\prime}+y=0$ about $x=0$ using Frobenius 07 method.

